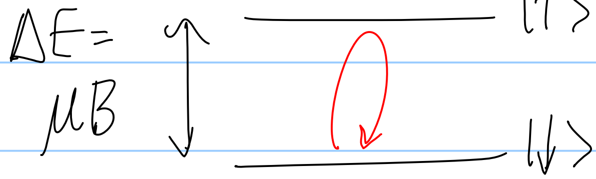
2-level systems

PHYS514

Note Title 2/9/2005

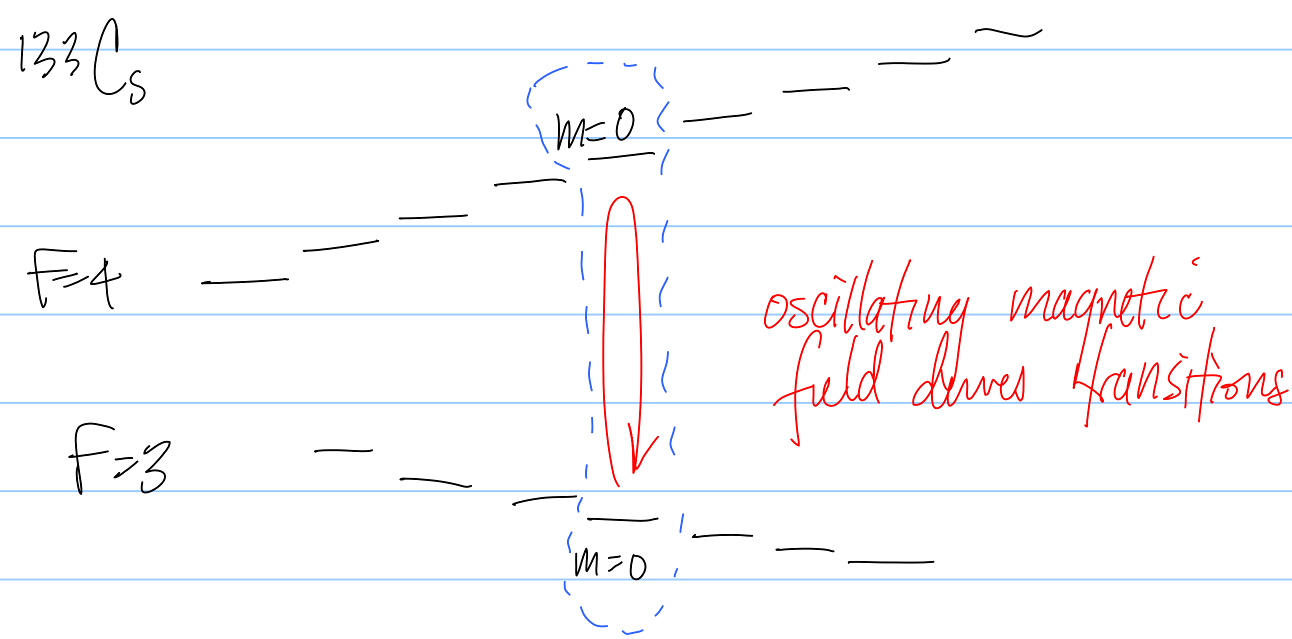
Examples

① Electron or proton spin is an ideal 2-level system!



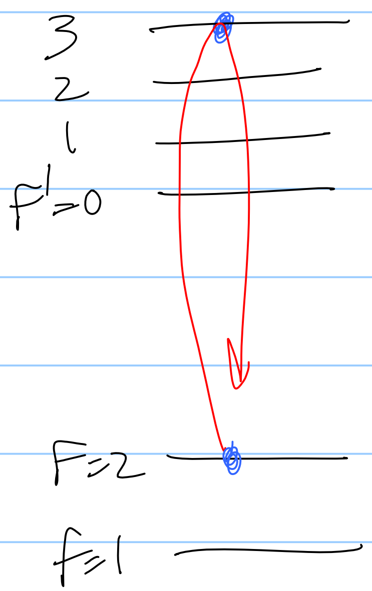
oscillating magnetic field will drive transitions

② hyperfine states + some tricks (used in an atomic clock)



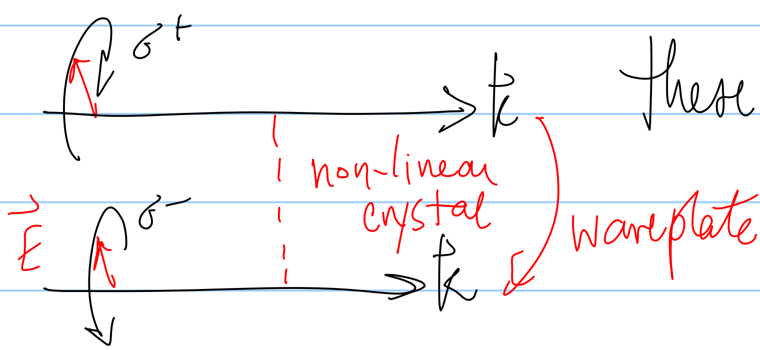
- we have to use tricks to restrict the Hilbert space to only two Zeeman levels
- these tricks can't be perfect — off-resonant excitation can play a role

③ atomic ground and excited states



- light will couple these states
- need tricks to keep $F=2, F'=3$ manifold closed
- watch out for off-resonant excitation again!

④ Photon helicity



these states are degenerate!

Let's tackle this problem in the following order:

I) Spin-$\frac{1}{2}$ terminology

II) 2-state system with static coupling

III) driven 2-state system

- Bloch-vector picture
- Dressed-state picture

Then we can talk about Rabi oscillations, Ramsey experiments, spin-echo, resonance fluorescence ...

I learned this subject in the reverse order, and was confused for a long time!

I Spin-1/2 terminology — start by considering a real, spin-1/2 particle as a prototypical 2-level system

state kets $|+\rangle, |-\rangle$

spinor: $\chi = \begin{pmatrix} c_+ \\ c_- \end{pmatrix} = c_+|+\rangle + c_-|-\rangle$

we typically think of $|+\rangle$ and $|-\rangle$ as "up" and "down" states with respect to a z-axis

$S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{\hbar}{2}\sigma_z$ here $\sigma_x, \sigma_y, \sigma_z$ are Pauli matrices

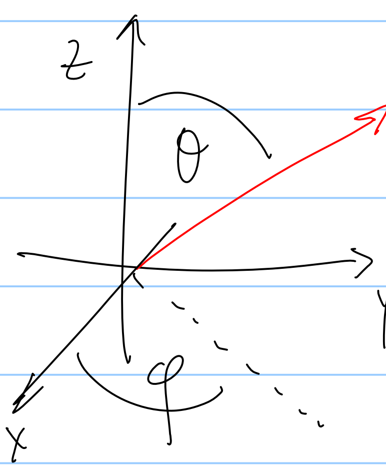
$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{\hbar}{2}\sigma_x$$

$$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \frac{\hbar}{2}\sigma_y$$

<u>any spinor can be written as:</u>

$$|\chi\rangle = \cos\frac{\theta}{2}|+\rangle + e^{i\varphi}\sin\theta/2\,|-\rangle$$

<u>in a rotated basis:</u>



$$|+\rangle_{z'} = \cos\frac{\theta}{2}e^{-i\varphi/2}|+\rangle + \sin\frac{\theta}{2}e^{i\varphi/2}|-\rangle$$

$$|-\rangle_{z'} = -\sin\frac{\theta}{2}e^{-i\varphi/2}|+\rangle + \cos\frac{\theta}{2}e^{i\varphi/2}|-\rangle$$

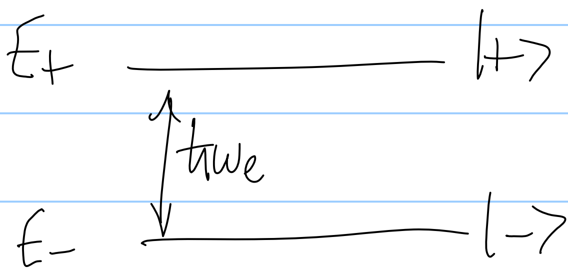
<u>Note:</u> rotation operator: $e^{-i\vec{\sigma}\cdot\hat{n}\,\varphi/2}$

<u>What does a spin-1/2 particle do in a uniform field?</u>

$H = -\vec{\mu}\cdot\vec{B}$ take $\vec{B}$ along $\hat{z}$, assume $\mu < 0$

$$H = \omega_\ell S_z = \frac{\hbar\omega_\ell}{2}|+\rangle\langle+| - \frac{\hbar\omega_\ell}{2}|-\rangle\langle-|$$

$$\omega_\ell = \frac{2\mu B}{\hbar}$$ Larmor frequency



What if I add a static, transverse field?

we call this adding a "coupling", where $B_\perp$ is the coupling between $|+\rangle$ and $|-\rangle$

$$\vec{B} = B_0 \hat{z} + B_\perp \hat{x}$$

$$H = -\vec{\mu} \cdot \vec{B} = \frac{\hbar \omega_e}{2} S_z + \frac{\hbar \omega_e}{2} \frac{B_\perp}{B_0} S_x$$

$$H = \frac{\hbar \omega_e}{2} \begin{pmatrix} 1 & B_\perp/B_0 \\ B_\perp/B_0 & -1 \end{pmatrix}$$

now, $\omega_e = \frac{2\mu B_0}{\hbar}$

we often think about $B_\perp$ as being small, but we're going to do an exact solution!

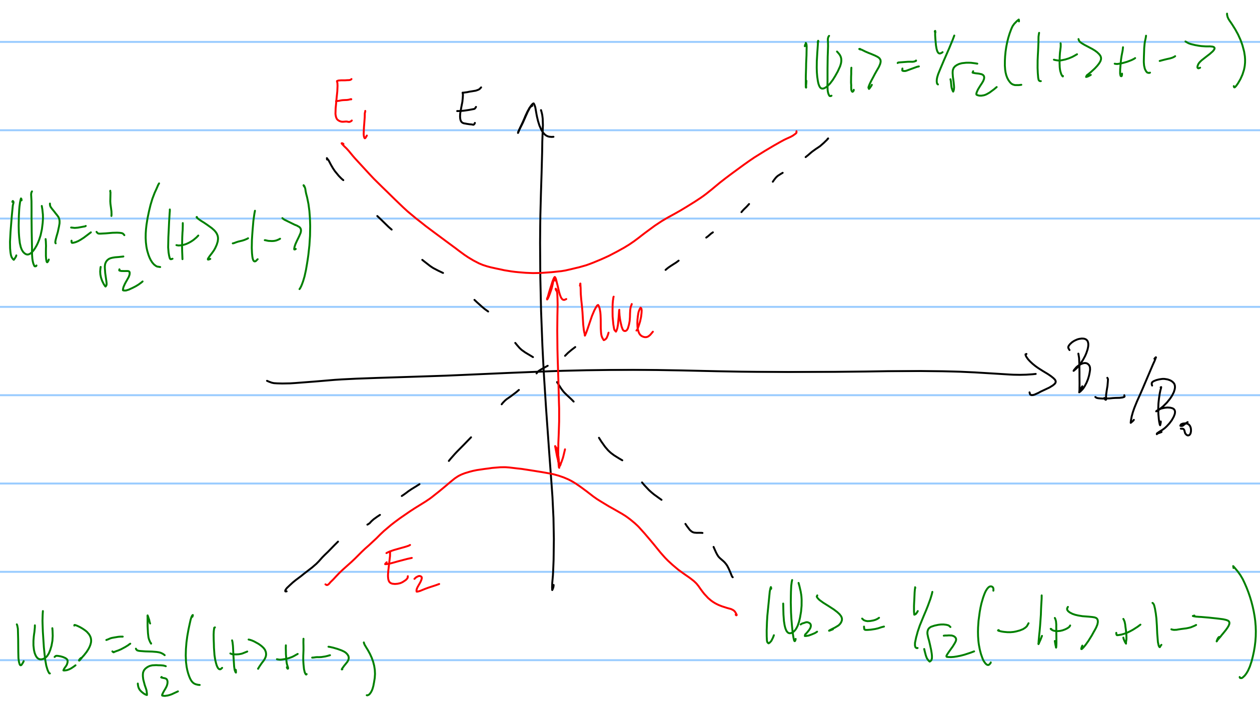
let's diagonalize H:

*be careful with my notation compared w/ Claude's — we, starting now, have different conventions

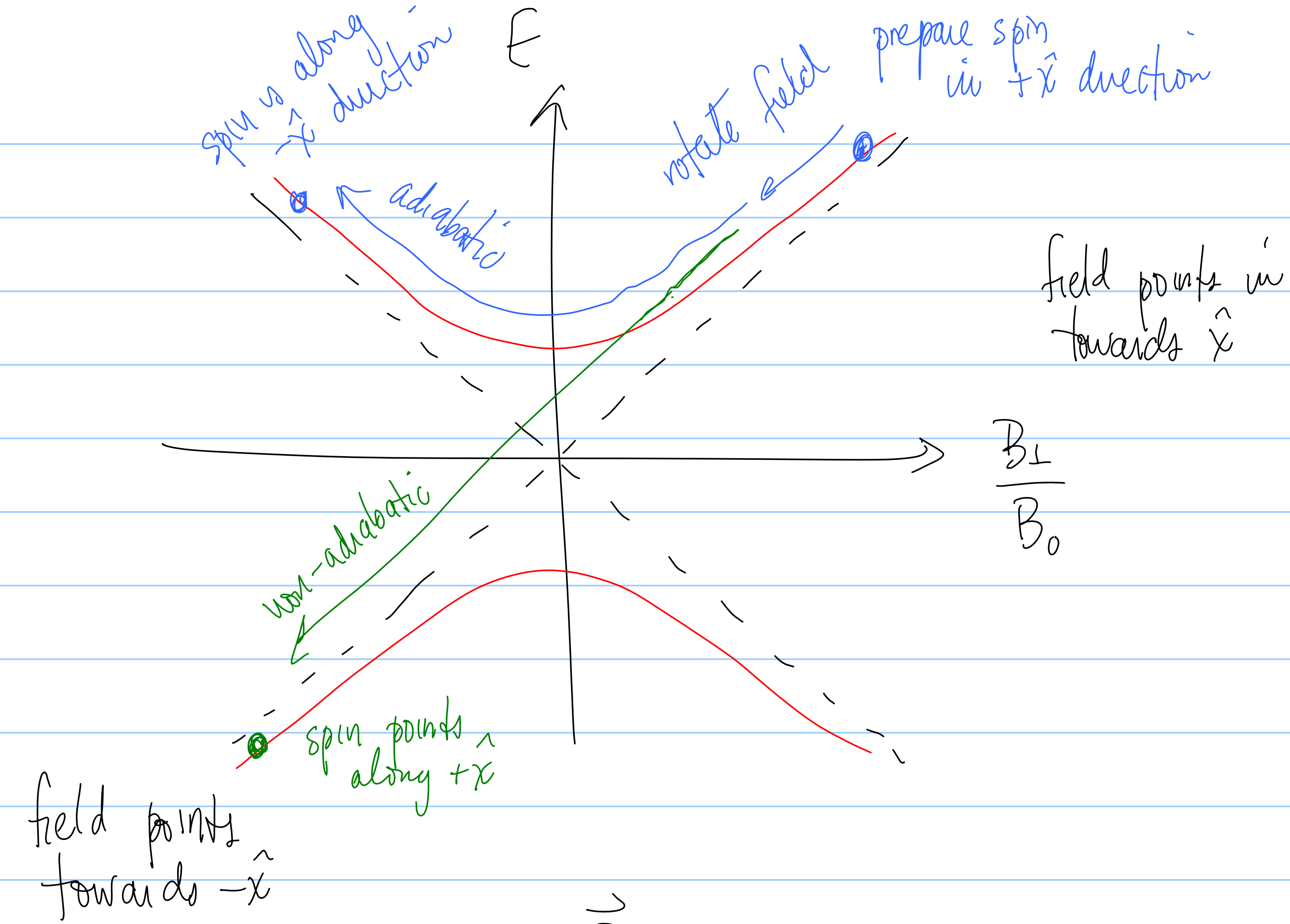
$$E_1 = +\frac{\hbar\omega_e}{2}\sqrt{1+\left(B_\perp/B_0\right)^2}$$
$$E_2 = -\frac{\hbar\omega_e}{2}\sqrt{1+\left(B_\perp/B_0\right)^2}$$
new energies!

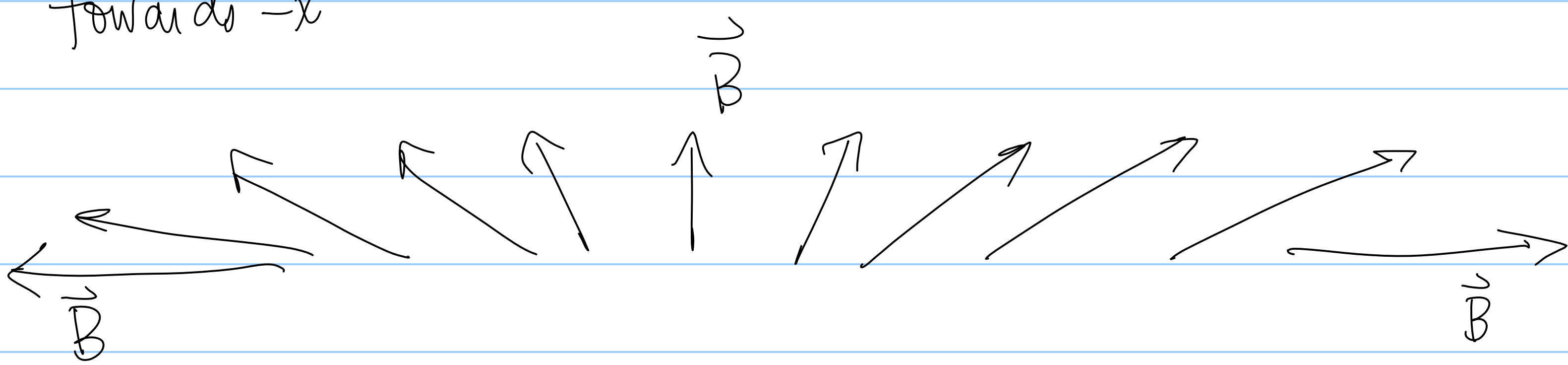
$$|\psi_1\rangle = \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}|-\rangle$$
$$|\psi_2\rangle = -\sin\theta/2\,|+\rangle + \cos\frac{\theta}{2}|-\rangle$$
new eigenstates!

where $\tan\theta = \dfrac{B_\perp}{B_0}$



All the physics of Majorana transitions is in this diagram!





Using time-dependent theory, the diabatic transition probability can be solved exactly ... see also the Landau-Zener formula for a generic, approximate solution:

$$P_D = e^{-2\pi\Gamma}$$

$$\Gamma = \frac{a^2}{\hbar|\alpha|}$$

$$a = \hbar\omega_e$$

$$|\alpha| = \left|\frac{\partial}{\partial t}(E_1 - E_2)\right|$$